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The Cyclical Advancement of Drastic Technologies

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Drastic technological changes are cyclical because basic R&D is carried on only at times when entrepreneurial profits for incremental technologies of the prevailing technological paradigm fall close to zero. The model is essentially an endogenous technological change framework. Varieties, input to the final good production, are composite goods. Each composite good is produced by a set of intermediaries, outgrowths of basic R&D and applied R&D. The basic intermediate, product of basic R&D, is modeled as in Romer (1990). Complementary intermediates, the outgrowths of applied R&D, do show the property of falling profits. The falling character of profits implies that basic R&D becomes more yielding than applied R&D at certain points in time. Research people switches back and forth between the applied and basic research sectors, creating (endogenous) cycles in the advancement of drastic technologies and economic activity.

Keywords: GPT; growth cycles; basic R&D; applied R&D; economic growth

JEL Codes: O11, O31, O40.

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1. Introduction

It has been first debated by Kondratieff (1926) that capitalism has long waves, regular fluctuations in economic life with a wavelength of 45-60 years. Schumpeter (1939) proposed that the cause of long run cycles might involve *discontinuities* in the process of drastic technical innovation. Historical evidence indeed indicates that neither production nor technological progress is a smooth process, and that major innovations *tend to* appear in clusters in certain periods (Olsson, 2001; Gordon, 2000; Mokyr 1990; Kleinknecht, 1987; van Duijn, 1983; Mensch, 1979).

Given the significant effect of technological change on economic growth (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992), a better understanding of the reasons behind the cyclical evolution of output and technology may have strong policy implications. In particular, smoothing out the cyclical advancement may bring about improvement in the long run performance of an economy.

Surprisingly, the clustered appearance of drastic technologies has not received much attention from growth theory. Relatively recently, David (1990) and especially Bresnahan and Trajtenberg (1995) have made the term general-purpose technology (GPT) popular to the growth theory. The main aim of this literature is to emphasize the difference between drastic technologies and incremental technological changes in terms of their growth implications. Currently, the focus seems to be on whether an economy experiences a slowdown at the onset of a new technological change due to reallocation of resources from the old to the new sectors or not (see several chapters in Helpman, 1998). Hence, the focus seems to be on temporary cycles that may be created by new technological paradigms at the onset of their introduction to the economy.

The aim of this study is to show why drastic technological change tends to proceed in a cyclical fashion. We conjecture that the main factor behind observing that drastic technological changes appear in clusters is eventually exhausting profit opportunities in incremental technologies of the existing technological paradigm.

Our model is essentially an extension of Romer (1990). The model consists of two R&D-sectors, labeled basic and applied, which respectively generate basic innovations for basic intermediary sector and applied innovations for complementary intermediate sectors. In particular, we suppose that each new drastic technology leads to emergence of *one* basic intermediary good and n complementary intermediary goods. These $n+1$ intermediaries are used in the production of a composite good, which becomes a variety in the production of final good. Indeed, each new composite good pushes upward the production frontier of the final good. There are two types of inputs in the model, physical capital and labor. Labor is further divided into three types, namely unskilled labor, skilled labor, and research labor, each of which is demanded only in one sector: unskilled labor enhances final good production (together with composite good varieties), skilled labor is used in the production of complementary intermediate sectors, and research labor is employed in R&D sectors. Finally, capital is used in the production of basic intermediate in the form of foregone output.

A good example to the exercise that we advance here is perhaps the computer. Suppose that the microprocessor represents the GPT (basic technology) and hardware and software are outgrowth of applied technology. Producers of intermediaries, each a monopolist, purchase patents of these technologies. The basic intermediate sector uses capital to produce microprocessors and the complementary intermediaries use skilled labor to produce the hardware and software. The computer, the outgrowth of assembling the microprocessor, the hardware, and the software, is a composite good and a variety (input) in Gross Domestic Production (GDP).

The critical contribution of the model is its success in generating declining profits among n “varieties” in the complementary sector. Positive monopoly profits of intermediate sectors are transferred to R&D people in the form of wages (cf. Romer, 1990). Researchers exploit positive profit opportunities of a prevailing technological paradigm by making incremental, non-drastic innovations. As profit opportunities become exhausted, it becomes more yielding to invest in a new technological paradigm at a certain point. Researchers then switch to work on the next drastic innovation (technological paradigm). Incremental innovation resumes within the new paradigm and endures until profit opportunities fall close to zero again. Thus, drastic technological change and economic development proceeds in long waves.

The model contributes to the (growth) literature in several ways. First, it develops a formal model of a mechanism that creates endogenous fluctuations in economic activity. Second, it demonstrates a way to introduce asymmetry in the intermediate market, which is rarely done in the literature.¹ This paper shows that asymmetric profit opportunities in the intermediate sector(s) are a lot more than a detail. Indeed, our paper shows that the falling character of these profits is the genuine source of economic fluctuations. Third, the model contributes to the literature by elaborating the causes of a possible slowdown at the onset of a new GPT. Last but not least, our model elaborates the role of basic and applied R&D mechanisms in the growth process. It shows that the impact of these two R&D sectors in the long run growth process is significantly different.

The organization of the paper is as follows. The next section introduces the model in its basic form, and solves the model at long run equilibrium. It is shown that profits are falling to zero across the varieties in the complementary intermediates sector. The third section looks at the sequence of equilibrium points generated in the model. For matter of convenience, heuristically speaking, we label it as the ‘very’ long run analysis. This section shows that exhausting profits in complementary intermediates sector are the source of fluctuations in economic activity. The last section summarizes findings with concluding remarks.

2. The Model

Let us suppose that the final good Y production technology is represented by

$$Y = L^{1-\beta} \sum_{i=1}^B z_i^\beta \quad 0 < \beta < 1 \quad (1)$$

¹ To our knowledge, van Zon and Yetkiner (forthcoming) is the only work studying asymmetric

where L represents unskilled labor used in the production of GDP, z_i is a *composite good* each of which is produced by $n+1$ intermediaries, $1-\beta$ is the partial output elasticity of unskilled labor, and i is the index of technological paradigm (GPT). The higher the i , the more recent the GPT that a composite good (or any other variable) is associated. Our motivation for introducing a vector of composite inputs rather than of single inputs, as many endogenous technological change models do, is that basic and applied R&D sectors generate two substantially different sets of intermediate goods in the model. As usual, we assume that the final-good sector is a perfectly competitive market.

We suppose that the composite good production technology has a Cobb-Douglas representation. We conjecture that the following function generates a composite good:

$$z_i = \prod_{j=0}^n (x_{ij})^{\alpha_j} \quad \forall i = 1, 2, \dots, B; \quad \forall j = 0, 1, \dots, n; \quad \sum_{j=0}^n \alpha_j = 1, \quad \alpha_j > 0 \quad (2)$$

In equation (2), n is a large positive integer indicating the number of intermediaries that the i^{th} composite good is made up of and identical across the composite goods. x_{ij} is the j^{th} intermediary used in the production of the i^{th} composite input, and α_j indicates the *relative share* of j^{th} input in the total product of composite good z_i . We make a set of assumptions. First, we assume that $\alpha_{ij} = \alpha_{i'j}$ for $\forall i, i' \in 1, 2, \dots, B$, which implies that we can omit subscript i from now on in α_{ij} . Second, we assume that *complementary* intermediates are ranked such that $\alpha_j > \alpha_{j'}$ if $j < j'$, $\forall j, j' \in 1, 2, \dots, n$. This assumption is however not restricting since it is matter of reordering in a Cobb-Douglas technology. It is worth to note two things in this assumption. First, we do not impose any condition on the ordinal value of α_0 . Second, the assumption contains that $\alpha_j \neq \alpha_{j'}$, that is, none of any pair of $(\alpha_j, \alpha_{j'})$ are alike. Third, we assume that α_n is at the *neighborhood* of zero. Given that (i) n is a large number, (ii) α_j are in a descending order, and (iii) the

intermediate sectors in endogenous technological framework.

sum of α_j is one, it is not imperceptive to assume that α_n is at the neighborhood of zero. The intuition behind this interpretation will be clear as we progress. Fourth, for matter of simplicity, we associate subscript 0 with the core intermediary good and $1, 2, \dots, n$ with the complementary intermediaries.² Hence, α_0 and $\alpha_1, \dots, \alpha_n$ are interpreted as respective relative shares of these two types of intermediate goods in total product of a composite good. Assuming a single core intermediary is solely for matter of tractability. From now on, we shall use 0 and j to designate the core intermediate and complementary intermediate related variables and parameters, unless otherwise stated. The evolvement of long run equilibrium is described below.

The Final Good Sector

A representative firm's profits are

$$\Pi_Y = L^{1-\beta} \sum_i z_i^\beta - \sum_i p_i z_i - wL \quad (3)$$

where composite output price is normalized to one, L is unskilled labor used in the production of final good, p_i is the user cost (price) of the composite input, and w is the rental price of unskilled labor. First order conditions with respect to z_i and L are

$$p_i = \beta L^{1-\beta} z_i^{\beta-1} \quad (4)$$

$$w = (1-\beta) L^{-\beta} \sum_i z_i^\beta \quad (5)$$

Note however that neither equation (5) is standard unskilled labor demand function nor is equation (4) an (inverse) input demand function. At least, not yet in their explicit form. In order to find out the explicit labor demand function, and input demand

² Complementary intermediaries can be associated with “innovational complementarity” character of

functions for intermediaries x_{i0} and x_{ij} , we must first associate the first order conditions of the final good market to the composite good production technology.

One way to link the final good sector to intermediary markets is to use cost minimization. Let us suppose that the intermediary-good prices are denoted by $(q_{i0}, q_{i1}, \dots, q_{in})$, in which the first price is associated with the core sector, x_{i0} , and others are associated by the complementary sector, (x_{i1}, \dots, x_{in}) . Then, total cost corresponding to the composite good i is $C_i = \sum_{j=0}^n q_{ij} x_{ij}$. Minimizing total costs subject to equation (2) yields

$$q_{ij} \cdot x_{ij} = \lambda_i \alpha_j z_i \quad j \in 0, 1, \dots, n \quad i \in 1, 2, \dots, B \quad (6)$$

Note that summation of equation (6) over j gives $C_i = \lambda_i z_i$, that is, the cost of producing the composite intermediate z_i is shadow price of composite input times quantity. Hence, λ_i works also as unit-price p_i of composite input i .

Substituting the optimum condition for the j^{th} intermediate of the i^{th} GPT, x_{ij} , from equation (6) into equation (2) gives

$$\lambda_i = \prod_{j=0}^n \left(\frac{q_{ij}}{\alpha_j} \right)^{\alpha_j} . \quad (7)$$

Equation (7) shows that the shadow price of the i^{th} composite input, λ_i , is a kind of geometric average of intermediate-good prices weighted by their respective input shares. Note that equation (7) is straightforward extension of two-input cost minimization problem under Cobb-Douglas technology.

GPTs as advanced by Bresnahan and Trajtenberg (1995).

Using equations (4) and (7) in equation (6) gives the inverse input-demand function for any intermediate good

$$x_{ij} = \left(\frac{\alpha_j}{q_{ij}} \right) \beta^\sigma L \prod_{k=0}^n \left(\frac{\alpha_k}{q_{ik}} \right)^{\alpha_k (\sigma-1)} \quad (8)$$

where $\sigma = 1/(1 - \beta)$ is the inverse of partial output elasticity of unskilled labor. As there are two types of intermediary goods for any GPT, we must consider them apart.

The Basic-Intermediate Sector

Let us first consider the core sector, indexed by 0. The derived demand function of the core sector, x_{i0} , by using equation (8), is

$$x_{i0} = \left(\frac{\alpha_0}{q_{i0}} \right)^{1+(\sigma-1)\alpha_0} \beta^\sigma L \prod_{k=1}^n \left(\frac{\alpha_k}{q_{ik}} \right)^{\alpha_k (\sigma-1)} \quad (9)$$

As equation (9) indicates, x_{i0} is inversely related with its own price. Throughout this study, we assume that prices of other intermediate goods (complementary goods in this case) do not have any (cross) price effect.

We shall continue to handle the core sector's profit maximization problem à la Romer (1990). At cost of some repetition, we would like to show derivations. There is a monopolist holding patent rights of the basic intermediate associated with a GPT. The profit equation of any intermediate firm in the complementary sector is

$$\pi_{i0} = q_{i0}x_{i0} - r\eta x_{i0} \quad (10)$$

where it is assumed that each unit of production of x_{i0} uses ηx_{i0} units of resources in terms of foregone output. Profit maximization leads to the well-known markup over unit cost pricing condition:

$$q_0 = r\eta \frac{\varepsilon_0}{1 + \varepsilon_0} = r\eta\phi_0 \quad (11)$$

In (11), $|\varepsilon_0| = 1 + (\sigma - 1)\alpha_0 > 1$ is the own price elasticity of input x_{i0} , and markup rate ϕ_0 is greater than one ($\phi_0 > 1$). It must be noted that the price of the core-intermediate is symmetric along ‘generations’ only if the rental cost of capital r is identical along the generations. Indeed, we assume that capital is putty-putty, and the supply of capital (in the form of forgone output) is infinite at the interest rate r and therefore it is constant and identical in the sequence of long run equilibrium points, which is consistent with stylized facts of growth. Finally, we note that there is an inverse hyperbolic relationship between ϕ_0 and α_0 such that ϕ_0 is monotonically declining in α_0 , i.e., $\partial\phi_0/\partial\alpha_0 < 0$.

It is *not* yet right to substitute the core sector input price (cf. equation (11)) into the respective demand (c.f. equation (9)) in order to solve the equilibrium demand for x_{i0} because we need to determine first input prices in the complementary sector before proceeding further. The next step does this.

The Complementary-Intermediate Sector

The complementary sector works as follows. When a GPT and the basic intermediate of that drastic technology appear in the market, the idea but the patent is a public good. If profit opportunities in the intermediate market are sufficiently high, then blueprints of complementary goods will be developed by the applied R&D sector. Using these blueprints, monopolists of the intermediate sector produce complementary intermediates.

We assume that the main input in the production of complementary goods is skilled labor, H . For simplicity, we assume that one unit of skilled labor produces one unit of complementary-intermediate:

$$x_{ij} = h_{ij} \quad j = 1, \dots, n \quad (12)$$

where h_{ij} is the amount of *skilled labor* used in the production of intermediate good x_{ij} . Since there is perfect factor mobility across complementary sectors within each GPT and across GPT sectors, there must be a single factor price w_h in the complementary sector. Following the same line of reasoning as we did for the basic intermediary sector, profit maximization leads to

$$q_{ij} = w_h \phi_j \quad j = 1, \dots, n \quad (13)$$

where $\phi_j = 1 + \frac{1}{(\sigma-1)\alpha_j}$. As in the core-intermediate sector, the inverse hyperbolic relationship between ϕ_j and α_j holds. Since there are n complementary inputs, where n is a very large number, we may also consider plotting these n markup rates against their corresponding input shares. Then, we find out that there is an inverse relationship between the “order of appearance” and the markup rate. An economic explanation may go as follows. Recall that we supposed $\alpha_j > \alpha_{j'}$ if $j < j'$ and that α_n is at the neighborhood of zero. Consider now x_n . Its relative input share in total product of composite input is at the neighborhood of zero but it is marginally the most critical input in the sense that the production of the composite good is impossible without it, though all other core and complementary inputs could have been produced. In other words, relatively speaking, x_n has the highest importance among all complementary intermediates in the production of the composite good. Therefore, the markup over unit cost is the highest, though it is the last in the order of appearance.

Using (13) in (8) gives³

$$x_{ij} = \left(\frac{\alpha_j}{\phi_j} \right) \beta^\sigma L \left(\frac{\alpha_0}{q_0} \right)^{\alpha_0 \beta \sigma} (w_h)^{-[1+(1-\alpha_0)\beta\sigma]} \prod_{k=1}^n \left(\frac{\alpha_k}{\phi_k} \right)^{\alpha_k \beta \sigma} \quad (14)$$

Equation (14) shows the inverse relationship between demand for any intermediate and the rental costs of inputs in the production of intermediaries, where equation (11) defines q_0 .

Recall that we assumed the use of skilled labor is limited to complementary sector. Under this assumption, for given supply, it is straightforward to calculate ‘sector-specific’ rental price of skilled labor w_h . This will be our starting point to solve the model at long run equilibrium.

Long-run Equilibrium

Let us suppose that we are at long-run equilibrium, the state that a cluster of new composite goods (a cluster of basic intermediates together with their complementary inputs) has been just added to the production frontier. Then, the demand-supply equilibrium of skilled labor in the complementary sector would be

$$H = \sum_{i=1}^B \sum_{j=1}^n h_{ij} = \sum_{i=1}^B \sum_{j=1}^n x_{ij} \quad (15)$$

Using (14) in (15) gives the equilibrium wage rate for skilled labor for given H , L , and r :

³ Note that $\sigma - 1 = \beta\sigma$.

$$w_h = \beta^{\sigma\chi} L^\chi \left(\frac{\alpha_0}{q_0} \right)^{\alpha_0 \beta \sigma \chi} \left(\frac{G_2 B}{H} \right)^\chi G_1^\chi \quad (16)$$

where $\chi = \frac{1-\beta}{1-\alpha_0\beta}$, $G_1 = \prod_{k=1}^n \left(\frac{\alpha_k}{\phi_k} \right)^{\alpha_k \beta \sigma}$, and $G_2 = \sum_{k=1}^n \left(\frac{\alpha_k}{\phi_k} \right)$. Note that $0 < \chi < 1$ due to the fact that $\alpha_0\beta < \beta$. Properties of G_1 and G_2 are as follows. First, G_1 and G_2 are constants and does not change along GPTs (hence, we may guess that they will not play any role in the generation of sequence of equilibrium points). Second, G_1 and G_2 are less than one. To see this, note that $\sum_{j=1}^n \alpha_j = (1-\alpha_0)$, by definition. Then, given the fact

that $(\alpha_j / \phi_j) < \alpha_j$, then, necessarily, $\sum_j (\alpha_j / \phi_j) < (1-\alpha_0) < 1$ and $\prod_{j=1}^{n_i} \left(\frac{\alpha_j}{\phi_j} \right)^{\alpha_j} < 1$.

Third, given the fact that $\left(\frac{\alpha_j}{\phi_j} \right)^{\alpha_j} < \left(\frac{\alpha_j}{\phi_j} \right)$ for any α_j , it is always true that $G_1 < G_2$.

Several observations concerning equation (16) are in order. First, skilled labor wages increase as the stock of GPTs rises for given L , H , and r . This is a ‘normal’ result in the sense that, as new GPTs are introduced, more intermediaries use the same (given) resource. Second, an increase in H or a decrease in L will lower skilled wages. An (exogenous) increase in the supply of skilled labor will certainly has a direct impact on its own price. The latter is the result of a rather indirect mechanism. A decrease in L lowers the ‘demand for composite inputs’ due to lower final good production. Consequently, the demand for complementary inputs is undercut and hence wages for skilled labor decreases.

The equilibrium price of a complementary product q_j mimics the skilled labor wage (cf. equation (13) and (16)). One interesting characteristic of complementary-goods prices is that they are “asymmetric” along varieties within a GPT because q_j is a function of input-share parameters. More particularly, $q_j < q_{j'}$ as $\alpha_j > \alpha_{j'}$ for $j < j'$. Thus, ‘older’ complementary intermediates charge lower prices. The economic intuition is very clear: The complementary sectors that have higher input shares in total product

of the composite good face higher price elasticities. Therefore, relatively speaking, they have to charge lower prices for their intermediaries to exploit positive profit opportunities of their product.

The equilibrium value of each complementary intermediate can be calculated by using equations (14) and (16):

$$x_j = \left(\frac{\alpha_j}{\phi_j} \right) \frac{H}{BG_2} \quad (17)$$

Three characteristics of equation (17) are in order. First, equilibrium values of intermediaries are dissimilar within a GPT (but identical along GPTs). The first term in the parenthesis on the right hand side of the equilibrium is the source of asymmetry across complementary goods. Second, the equilibrium value declines with α_j . It is straightforward to see this result by checking $\frac{\partial(\alpha_j / \phi_j)}{\partial \alpha_j}$, which is positive. In other words, the later the intermediate appears in the market, the less its equilibrium value. Third, for given H (B), B (H) is associated negatively (positively) with x_j .

The profits of the j^{th} firm in the i^{th} GPT (in the complementary intermediaries) is found by substituting the respective values of w_h and x_j from (16) and (17) in profit equation $\pi_j = (\phi_j - 1) \cdot w_h \cdot x_j$:

$$\pi_j = \left(\frac{1}{\sigma - 1} \right) \left(\frac{1}{\phi_j} \right) L^\chi \beta^{\sigma\chi} \left(\frac{\alpha_0}{q_0} \right)^{\alpha_0 \beta \sigma \chi} \left(\frac{H}{BG_2} \right)^{1-\chi} (G_1)^\chi \quad (18)$$

The most obvious characteristic of profits in equation (18) is its falling nature in input shares. Recall that we assumed α_j are ranked in a descending order. Thus, *the later the intermediate appears, the less the profit it earns*, according to equation (18). The

economic reasoning of falling profits is as follows. We discussed above that there is an inverse relationship between the order of appearance of an intermediate and its relative importance. This is because it is relatively easier to give up the production of the composite input at the ‘early’ steps of production. The later the intermediate appears, the higher is its relative ‘importance’ in the production of the composite input and hence it is more ‘costly’ to give up the whole production. Hence, the price, charged by a particular intermediate, is increasing as its “importance” in the production rises (cf. Equation (13)). Consistently, equation (17) shows that equilibrium quantities for intermediates are positively related with their order of appearance. ‘Early intermediates’ face more price-elastic demand and therefore charge lower prices to capture the entire profit opportunities specific to their products. Profits are outgrowth of (equilibrium) quantity and prices, given a unique rental price of skilled labor across the intermediates. We infer from equation (18) that the fall in the equilibrium quantity due to a decrease in the order outweighs the rise in prices and therefore profits decline in correspondence with the order of appearance in the market.

What is the importance of this finding? Under perfect foresight assumption, entrepreneurs in the complementary intermediate market would be aware of the *profit opportunities* of all intermediates 1 to n . Then, a monopolist would prefer to produce the intermediate that promises the highest profit opportunity among n varieties. Hence, the order of appearance of intermediates is function of the order of size of input shares. The assumption we made initially that input shares were ordered in a descending form was indeed an early indication of the market opportunities in the complementary sector.

Finally, we can calculate x_0 . Using equations (9) and (16), x_0 is⁴

$$x_0 = \beta^{\sigma\chi} \cdot L^\chi \cdot \left(\frac{\alpha_0}{q_0} \right)^{\sigma\chi} \cdot \left(\frac{H}{BG_2} \right)^{1-\chi} G_1^\chi \quad (19)$$

⁴ It is helpful to see (i) $-(1-\alpha_0)\beta\sigma = -((1-\chi)/\chi)$, (ii) $1 + \alpha_0\beta\sigma\chi = \sigma\chi$, and (iii) $1 - \alpha_0\chi = (1-\alpha_0)\sigma\chi$.

This is the equilibrium of x_0 . Note that x_0 implies the following equilibrium profit for the basic intermediate (cf. equation 10):

$$\pi_0 = r\eta \cdot (\phi_0 - 1) \cdot \beta^{\sigma\chi} \cdot L^\chi \cdot \left(\frac{\alpha_0}{q_0}\right)^{\sigma\chi} \cdot \left(\frac{H}{BG_2}\right)^{1-\chi} (G_1)^\chi \quad (20)$$

Following x_0 , π_0 are similar across the core sectors (i.e., along the GPTs).

As we now have all information concerning the composite good, we can proceed to find the equilibrium values of ‘aggregate variables’. Employing (17) and (19) in equation (2) gives us z_i . Using this value in (1), we can show that final output Y is⁵

$$Y = \beta^{\alpha_0\beta\sigma\chi} L^\chi \left(\frac{\alpha_0}{q_0}\right)^{\alpha_0\beta\sigma\chi} \left(\frac{H}{G_2}\right)^{1-\chi} (G_1)^\chi B^\chi \quad (21)$$

Equation (21) is not very much different than any reduced form final output production function but is richer. First, the “technological variety” variable B is the source of endogenous growth in the model, very much like the “love of variety” variable in Romer (1990). The basic difference is that B pushes the output frontier forward cyclically (that we will show in the next section). The fundamental similarity with the existing literature is that the growth rate of B is function of level of R&D people employed in the basic R&D (cf., the third section). Second, unskilled labor and skilled labor are (exogenous) sources of growth of output, if these variables are presumed to grow in time. Third, though applied R&D plays a critical role in terms of producing new composite varieties, it does not play any explicit role in the advancement of long run growth at equilibrium. Hence, our model suggests that we need to reach a better

⁵ We can calculate aggregate capital and check if the ratio of the two is constant, fitting to stylized facts. Aggregate capital is obtained by summing x_0 along the GPTs, $K = \eta \cdot \sum_i x_0$. It is straightforward to

understanding of elements that contribute to growth and development. Let us now look at the generation of the sequence of long run equilibrium points that we occasionally call it the ‘very’ long run.

3. The “Very” Long run

The sequence of long run equilibrium points is generated by R&D sectors in our model. This section is on how the mechanism works. We assume in this model that basic and applied research sectors use *research labor*, a special type of labor endowed with frontier knowledge, in generating blueprints. The two R&D sectors compete for the ‘scarce’ research labor in the model. We show that that competition is linked to falling profit opportunities in the intermediate market, and hence drastic technologies are advanced in clusters. Before starting to discuss how the model generates a sequence of equilibrium points, let us elaborate time in the model. The model uses three types of time. First, there is real time, denoted by s . Second, ω is used to denote applied R&D time. We will soon show that basic R&D and applied R&D do not take place simultaneously but one follows another under an endogenous switching mechanism. Furthermore, we will also show that applied R&D activities are neither a continuation of the applied R&D activities of the previous GPT nor are they continuous for the most recent GPT. Third, we index the time points that the model-economy realizes jumps in the drastic technology stocks by t and call it as ‘GPT time’. We illustrate the association of R&D activities in the real time line below:

show that $K / Y = \alpha_0 \beta / r \phi_0$. The ratio is constant for a constant r , which must be true, at least at long run.

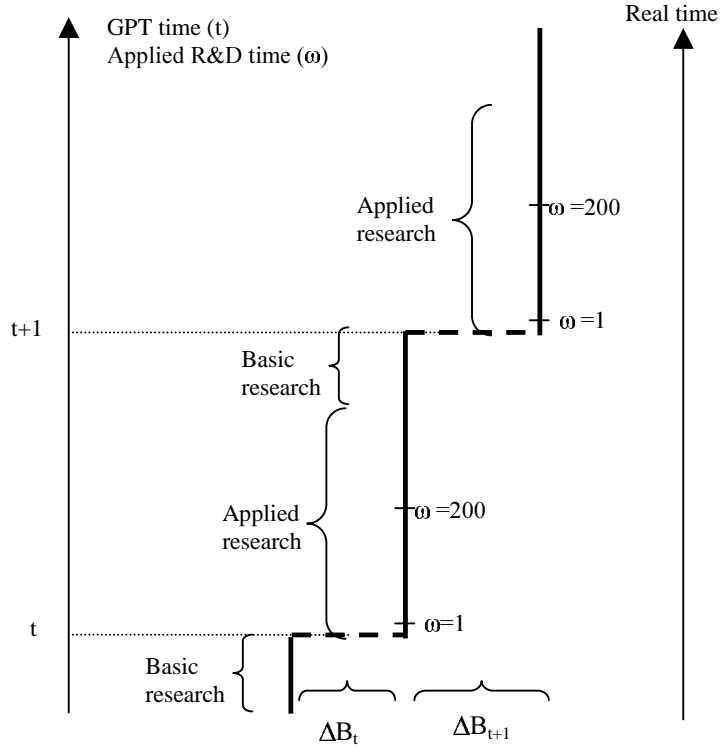


Figure 1 Associating inventions with real time

The Basic Research Sector

We conjecture that blueprints accumulate according to the following difference function:

$$B_{t+1} - B_t = \delta R_B B_t \quad (22)$$

where B is stock of GPT, δ represents the productivity of the blueprint generation process, and R_B is the amount of research people used in generating blueprints of GPTs. The way we defined the GPT generation mechanism is a simple difference equation (with only homogenous part) and its solution is $B_t = (1 + \delta R_B)^t$. The mechanism generates (discrete) perpetual growth. In particular, the stock of GPTs increase at increasing rates at equal time distances. This result can be rationalized by the public good character of ideas (cf. Romer (1990)).

As usual, whenever the basic R&D sector undertakes research, the proceeds of blueprints are paid as wages, w_0 . Suppose that B_t has been already invented (thus given). The profits for the next basic R&D activity $\pi_{t+1,0}$ would be

$$\pi_{t+1,0} = P_{t+1,0}(\delta R_B B_t) - w_{t+1,0} R_B \quad (23)$$

where $P_{t+1,0}$ is the price of designs, $w_{t+1,0}$ represents the rental rate of R&D labor for the blueprints developed, R_B is the amount of research people used in the invention process, subscript *zero* indicates that the variable is related to basic R&D, and subscript $t+1$ shows that drastic inventions are made between times $t+1$ and t . Equilibrium process yields

$$w_{t+1,0} = P_{t+1,0} \delta B_t, \quad (24)$$

a condition that must be satisfied when research staff is ever employed in the basic R&D. Note that the stock of B_t is taken as given as anyone engaging in basic research can freely take advantage of the entire existing stock of GPT blueprints. Next, we describe the employment condition in the applied sector.

The Applied R&D Sector

The dynamics of the applied R&D sector is substantially different than the basic sector though the blueprint accumulation function of the sector resembles of the basic R&D sector. The first reason to the difference is that applied R&D efforts of previous GPTs have no effect on the current applied R&D activities. Second, the applied R&D activities of the current GPT is discontinuous because complementary intermediate j has no relevance with j' , $j, j' \in 1, 2, \dots, n$. We conjecture the applied R&D accumulation function as follows:

$$n_{j,\omega+1} - n_{j,\omega} = \begin{cases} \xi B_t R_A & \text{if } n_j \leq B_{t+1} - B_t \\ 0 & \text{otherwise} \end{cases} \quad j = 1, 2, \dots \quad (25)$$

where n_j denotes the stock of applied technology for the j^{th} variety generated, ξ represents the productivity of the blueprint-invention process, B_t denotes the stock of GPT paradigms (including the most recent) that applied R&D sector enjoys freely, and R_A is the amount of research labor employed. Clearly, $R_A + R_B = R$, which is given in the model. Equation (25) says that the applied R&D activities will produce from each blueprint to the amount equal to the number of GPTs produced in the related GPT bundle. The blueprint generation mechanism in equation (25) is a simple difference equation (with homogenous and particular parts) and its solution is

$$n_{j,\omega} = \begin{cases} \omega \cdot \xi \cdot B_t \cdot R_A & \text{if } n_j \leq B_{t+1} - B_t \\ 0 & \text{otherwise} \end{cases} \quad j = 1, 2, \dots \quad (26)$$

given that $n_{j,0}$ is zero for all j . According to equation (26), blueprints accumulate as a linear positive function of the amount of research labor used as long as it is less than number of GPTs produced in the most recent basic R&D activity. For clarity, we would like to illustrate equation (26) with an example. Suppose that the economy has just

produced nine new GPT blueprints. Then, according to model, the applied R&D has to produce nine units of blueprints for the first ($j=1$) complementary input, nine units of blueprints for the second complementary good ($j=2$), and so on. Furthermore, suppose that the applied R&D sector can produce three units of blueprint per time in accordance with equation (26). Then, the graphical illustration of equation (26) would be as follows:

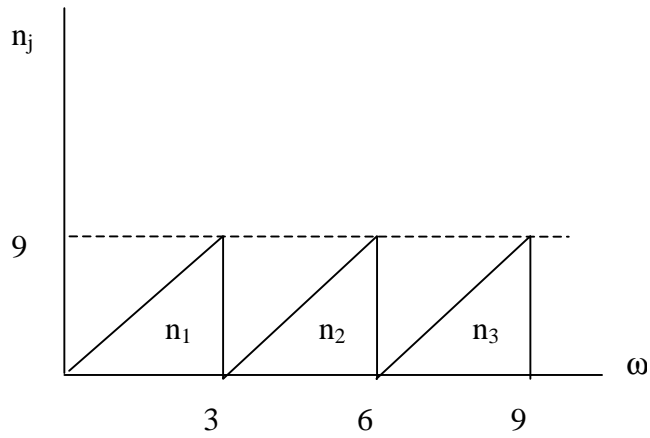


Figure 2 Blueprint accumulation in Applied R&D

The blueprint accumulation continues as long as the R&D people are employed in the applied R&D sector (the condition is given in the next subsection).

The profits of the j^{th} design will be $\pi_j = P_{t,j} \xi B_t R_A - w_{t,j} R_A$ and equilibrium process produces

$$w_{t,j} = P_{t,j} \xi B_t. \quad j = 1, 2, \dots \quad (27)$$

where $P_{t,j}$ is the price of the j^{th} complementary-good design, $w_{t,j}$ is the rental rate of R&D labor in the j^{th} design, and t indicates that the prevailing GPT bundle is created at times t and $t-1$. Equation (27) gives the wage rate w_j that the applied R&D sector must pay in order to undertake research in the sector.

The Cyclical Use of R&D-labor

The unit value of a new blueprint must be equal to present discounted value of profit stream generated in the intermediate sector, given that R&D sectors operate under perfect competition. The intuition is simple. Because the market for designs is competitive, the price for designs will be bid up until it is equal to the present value of the profit stream that a monopolist can extract. Hence, the price of each technology P_j $j=0,1,2..$ is equal to present discounted value of profit stream of the respective intermediary producer (cf. Romer (1990)). It is easy to calculate profit streams of intermediate sectors by using equations (18) and (20). Suppose that GPT bundle of time $t+1$ has already been invented at present. The present value of profits of the basic sector for any GPT in the *next* GPT-cluster would be

$$PV_{t+2,0} = \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \pi_{t+2,0} \Rightarrow$$

$$PV_{t+2,0} = r\eta \cdot (\phi_0 - 1) \cdot \beta^{\sigma\chi} \cdot \left(\frac{\alpha_0}{q_0} \right)^{\sigma\chi} \frac{G_1^{\chi}}{G_2^{1-\chi}} \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \frac{L^{\chi} \cdot H^{1-\chi}}{(B_{t+1})^{1-\chi}} \quad (28)$$

In Equation (28), s denotes the real time and τ indicates the present. We assume that the growth dynamics of L and H are known to the system. It is critical to note that equation (28) is derived at equilibrium, meaning that the present value of profits received by the basic-intermediate producer is calculated under the assumption that n complementary goods for each GPT in the new cluster have been produced. In other words, we are able to calculate the present value of profits at the next equilibrium point.

Similarly, the present value of the j^{th} complementary-good at time τ , where the latest GPT stock available at that time is B_{t+1} , will be

$$PV_{t+1,j} = \frac{1}{\sigma-1} \cdot \frac{1}{\phi_j} \cdot \beta^{\sigma\chi} \cdot \left(\frac{\alpha_0}{q_0} \right)^{\alpha_0\beta\sigma\chi} \frac{G_1^\chi}{G_2^{1-\chi}} \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \frac{L^\chi \cdot H^{1-\chi}}{(B_t)^{1-\chi}} \quad (29)$$

The most interesting property of discounted profit streams in (29) is its falling nature. In particular, $PV_{t+1,n}$ must be at the neighborhood of zero, given our assumption that the very last input share α_n is at the neighborhood of zero (i.e., $PV_{t+1,n+1} = 0$). Evidently, R&D people will stop working on the prevailing technological paradigm after producing the n^{th} blueprint under perfect foresight assumption.

Recall that profit streams are captured by R&D people, independent of whether they are employed at basic R&D sector or at applied R&D sector (cf., equations (24) and (25)). Then, the falling nature of profit streams (in the applied sector) must be also reflected in the wages of R&D people employed in the applied sector. In particular, wages received by the R&D people working in the applied R&D must be falling as new blueprints for intermediaries are produced. This characteristic of our model is indeed the heart of cyclical advancement of technologies and long run business cycles. We discuss below the mechanism in detail.

The research labor decides on the use of their labor by comparing the real wages offered by the two research-sectors at any time. Given the linear blueprint production functions, all R&D people will be employed in only one sector, that is, only corner solutions are viable in the model (clearly, linearity is only for stylistic purposes). Suppose now that the basic R&D sector has just used the whole research staff. In particular, suppose that we have just produced the blueprint (bundle) for the basic intermediate of GPT $t+1$. The question is whether they would switch to produce complementary intermediaries for this GPT or switch to work on new GPT bundle. In our set up, the following conditions must hold in order to make this switching viable:

$$\begin{aligned}
w_{t+1,1} &> w_{t+2,0} \\
w_{t+1,2} &> w_{t+2,0} \\
(\dots) \\
w_{t+1,n} &> w_{t+2,0}
\end{aligned} \tag{30}$$

Since the model is not able to show transitional dynamics, we can only produce calculable switching conditions ex-post, meaning that we can check these switching conditions at equilibrium points. Indeed, under perfect foresight, doing this is not controversial because those GPTs that do not convert into a composite good would have never been started. Equation (30) indicates that in order for a GPT bundle, say B_{t+1} , to be viable, the real wage offered by the applied R&D sector for the first complementary good must be higher than the wage rate offered by the basic R&D sector of the next GPT cluster. If this condition holds, then the entire research people will shift to applied research. The same condition must also hold for blueprints of intermediaries 2,3,... Nonetheless, there is always an end to this process. Indeed, our assumption that α_n is at the neighborhood of zero implies that $w_{t+1,n+1}$ would be zero and hence the condition for switching to the next GPT technology is always secured. It might be argued that the assumption that α_n is at the neighborhood of zero is too strong. However, it must be noted that the genuine generator of the switching mechanism is not that assumption but the fact that profits in the complementary sector has a falling nature. Assuming that α_n is at the neighborhood of zero only secures the constancy of number of varieties in the model.

It is worth to mention that the wages in the R&D sector also experiences cycles. From equation (30) above, we know that $w_{t+1,1} > w_{t+2,0}$ but wages decline (towards zero) as new intermediaries are produced. When the model-economy starts to produce the next generation GPT bundle, first research people's wages experience $w_{t+2,0}$ and next a jump to $w_{t+2,1}$, where the latter can be substantially greater than the former. Then, it starts to fall again. This mechanism creates cycles in R&D wages, and none of these

cycles necessarily produce similar wage rates as skilled labor, unskilled labor and the stock of GPTs increase over time.

Closure of the Model

In order not to complicate the model further, we assume that consumption is determined by an exogenous saving rate proportional to income (cf., Solow (1956):

$$C_s = (1 - s)Y_s \quad (31)$$

where C_t is consumption, and s is exogenous saving rate.

Dynamics of the Output and the Broader Concept of Output

Equilibrium output Y and the broader concept of output Q are two types of output in our model.⁶ The former is associated with long run equilibrium points and the latter refers to values of output in transition between equilibrium points. Up to now, we have discussed the dynamics of the equilibrium output. We showed that gross output realizes upward shifts at times that a new composite good is started to be produced. Evidently, output between equilibrium states is different than equilibrium points. Unfortunately, it is not easy to show the exact values of the broader concept of equilibrium. In this subsection, we will try to give a sense of it.

Suppose that the model economy has just realized Y_t . At the next real time, say $\tau + 1$, the economy will generate the next GPT bundle, $B_{t+1} - B_t$, and ΔB units of core intermediaries $x_{t+1,0}$, associated with the recent bundle. Clearly, the broader concept of output is *not* $Q_{\tau+1} = Y_t + x_{t+1,0} \cdot (B_{t+1} - B_t)$ because (i) all production activities of existing GPTs are affected inversely by addition of a new intermediary and (ii) skilled and unskilled labor are growing. Nevertheless, we can infer likely impacts of addition of

⁶ See Chapter 5 of Barro and Sala-i-Martin (1995) on the broader concept of output.

new intermediates to the system. It is straightforward to check from long run equilibrium solutions that an addition of a new component to the existing system means a decrease in the available input resources per item. We do not know the exact equation in transitional period but can guess that on the one hand Y_t would decline by introduction of $x_{t+1,0}$ while, on the other hand, an exogenous increase would be realized in skilled and unskilled labor, which would allow for an increase in Y_t . This increase may counterweigh the decline in input resources per item and hence the broader concept of output may increase. In conclusion, there may or not be a decrease in output at the onset of emergence of a new technological paradigm, depending on several factors. In figure 2 below, we speculate two alternative paths (represented by circled dots and squared dots) that illustrate two alternative scenarios.

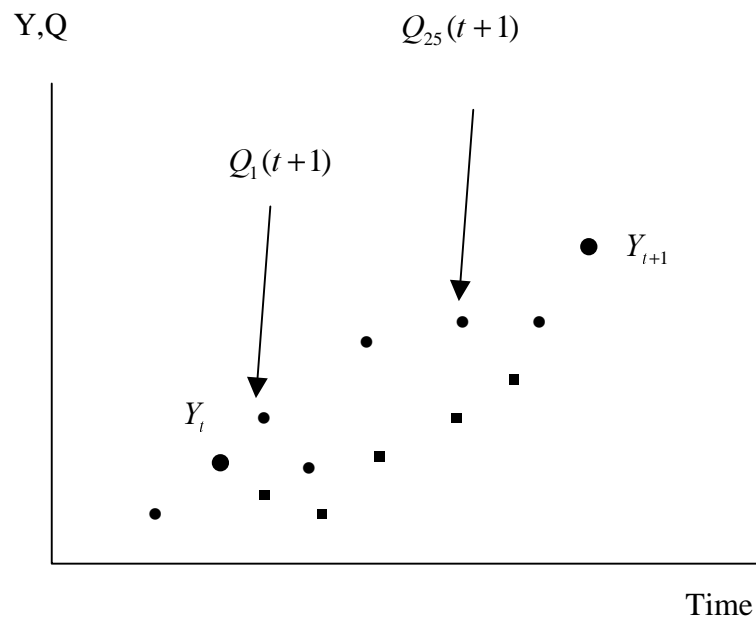


Figure 3 The Path of Output and Broader Concept of Output

In figure 3, the path formed by circled dots assumes that the pace of growth of inputs is sufficient to meet the additional demand created by the introduction of new intermediaries. The squared dots, on the other hand, illustrate a case that the economy realizes a fall at the onset of a new technology due to insufficient growth of inputs. Hence, our model shows that the current debate in GPT-growth literature on whether output declines at the onset of a new GPT is inconclusive, and the answer depends on the growth rate of inputs.

4. Conclusion

This study showed that exhausting profits in the incremental technologies with the existing technological paradigm could be the source of long run business cycles. New technological paradigms are advanced cyclically because R&D activities focus on the existing technological paradigm as long as there remain positive profit opportunities on it. Focus returns to basic R&D whenever the profit opportunities of the next bundle of drastic technologies are higher than that of the existing paradigm. Switching between the basic and applied technologies creates long run cycles in the economy. The paper showed also that temporary falls in growth at the onset of a new technological paradigm might be because the pace of growth of inputs was not meeting the additional resource needs created by the new paradigm.

This paper has many possible extensions. One of them is very exciting. The very existence of long run Kondratieff cycles brings into the scene the question of “are these cycles making ‘us’ better off or worse off? This is an interesting question because these cycles are created in response to market opportunities. Hence, if profit drives imply economic inefficiency in terms of welfare losses, there is a big room for policy intervention. This paper leaves this question open for future studies.

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